Heating Rate of the Driven Gas in a Hartmann-Sprenger Tube

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Theme

IT has been proposed by Phillips and Pavli[1] and more recently by Marchese et al.[2] to use the heat produced in a "Hartmann-Sprenger" (or HS) tube to ignite explosives. For this particular application, the heating rate of the oscillating gas is an important factor in determining the success of the technique. In this paper, the analysis of Ref. 3 is extended in order to calculate the heating rate of the oscillating gas in a HS-tube.

Contents

The HS-tube, in its original configuration, consists of a supersonic underexpanded jet driving oscillations in a cylindrical cavity whose axis coincides with the jet axis. It has often and erroneously been named "resonance tube."[4] Later, it has been shown that oscillations may also be produced with subsonic and supersonic, correctly expanded jets.[5-7] Sprenger[6] also discovered that considerable heating of the cavity can occur and reported temperatures in excess of 1000°C.

The dynamics of the HS-tube is described in detail in Ref. 7. Figure 1 represents a simplified wave diagram of the oscillation cycle. This cycle consists of four waves: an incident compression wave (i.c.w.), a reflected compression wave (r.c.w.), an incident expansion wave (i.e.w.) and a reflected expansion wave (r.e.w.). Also shown on the diagram is the entropy line, or contact front (c.f.), that separates the driver and the driven gas. It has been shown[7,8] that it takes only a few cycles for the oscillations to approach their full amplitude (limit cycle) once the driving jet starts exciting the cavity. Consequently, it is here assumed that the heating due to the various dissipation mechanisms starts only when the full pressure amplitude is reached. However, the starting of the oscillations is accompanied with an enthalpy increase of the driven gas which may be called "dynamical" heating. Once the limit cycle is reached, the ratio θ of the stagnation temperature T_{0_2} in the driven gas to the ambient temperature T_a is $[1+(\gamma-1)M_j^2]/[1+[(\gamma-1)/2]M_j^2]$ and is taken as the initial value of $\theta(=\theta_i)$ for dissipative heating computations.

Knowledge of the flowfield in a HS-tube enables one to calculate the power dissipated in the driven gas by shock wave irreversibilities and friction on the tube wall.[3] The power dissipated by the shock waves P_i may be written in the form $P_i/P_2=\phi_i\ [\gamma,M_2]$ where P_2 is the mechanical power transmitted to the driven gas, γ is the specific heat ratio and M_2 is the Mach number in field 2 of Fig. 1. Similarly, the power P_f dissipated by friction may be written in the form $P_f/P_2=\phi_f\ [\gamma,M_2,\ L/D,\ Re^*]$, where L/D is the length-to-diameter ratio of the tube and Re^* a Reynolds number. Re^* is defined as $D\,p_a/\mu_{0_j}(R\,T_{0_j})^{1/2}$ where p_a represents the ambient pressure, R the gas constant and μ_{0_j}, T_{0_j} the absolute viscosity and the stagnation temperature of the driving jet. The main heat evacuation mechanism is by a mass exchange that takes place at the contact front separating the hot and cold gases. The power P_m evacuated by this means is $P_m/P_2=\phi_m\ [\gamma,M_2,\ L/D,\ Re^*](\theta-1)\ \theta^{-1}$. The functions ϕ_i, ϕ_f and ϕ_m are given in Ref. 3. The other important heat evacuation mechanism is by forced convection on the inner wall of the tube. The power P_{cf_0} evacuated that way is $P_{cf_0}/P_2=\phi_{cf_0}\ [\gamma,M_2,\ L/D,\ Re^*]$ $(\theta-1)\ \theta^{-1}$. The function ϕ_{cf_0} is slightly different from the function ϕ_{cf} of Ref. 3 because in the initial phase of heating, owing to their thermal inertia, the tube walls remain essentially at ambient temperature. ϕ_{cf_0} may be written in the form

$$\phi_{cf_0}=0.046\left(\frac{\gamma}{\gamma-1}\right)(\gamma^{1/2}M_2Re^*)^{-0.2}Pr^{-0.6}\left(\frac{L}{D}\right)\left(1+\frac{\gamma-1}{2}M_2^2\right)I(M_2) \qquad (1)$$

where Pr represents the Prandtl number and $I(M_2)$ is the integral given in the Appendix of Ref. 3.

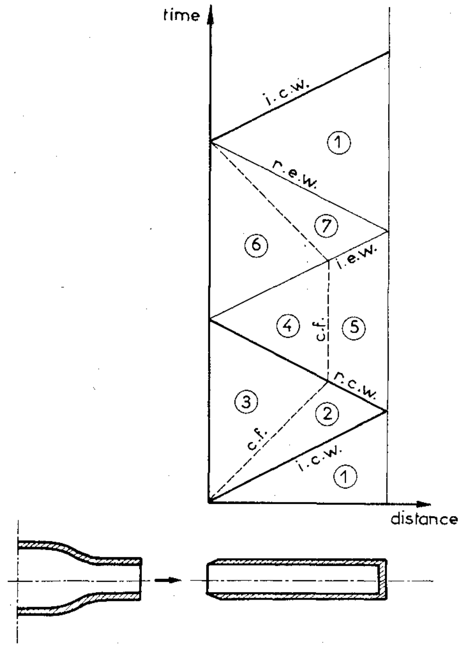

Fig. 1 Simplified wave diagram of the Hartmann-Sprenger tube.

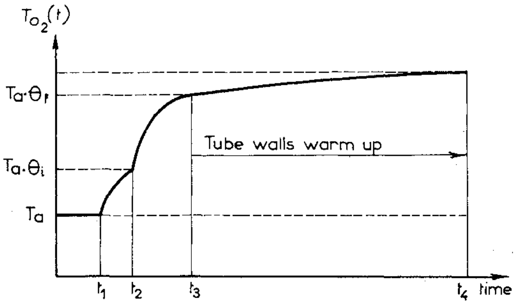

Fig. 2 Total temperature history of driven gas.

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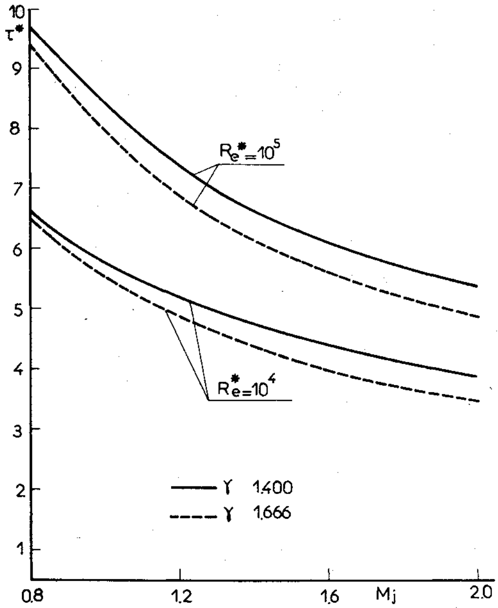

Fig. 3 Time τ^* taken by the driven gas to reach 99% of its total temperature increase ($L/D=33$).

The history of T_{0_2} is sketched in Fig. 2. Let t_1 denote the time at which the oscillations start. From t_1 to t_2, the "dynamical" heating of the gas occurs. From t_2 to t_3, the gas is heated by dissipative mechanisms, where the tube walls remain essentially at ambient temperature. From t_3 to t_4, the walls are gradually heated to an equilibrium temperature.

The heating rate of the gas from t_2 to t_3 is now considered. This rate is given by the difference between the heat produced and the heat removed at each cycle, that is

$$\rho_1 L\, S\, C_p \frac{dT_{0_2}}{dt} = (P_i + P_f) - (P_m + P_{cf_0}) \tag{2}$$

where ρ_1 represents the gas density in field 1 of Fig. 1, S the tube cross section, C_p the specific heat at constant pressure, and t the time. Introducing the dimensionless time $\tau = f_0 t$, where $f_0 = a_a/4L$ is the acoustical frequency of the tube at ambient temperature, the heating rate equation (2) may be written in the form

$$\frac{d\theta}{d\tau} = 4\left(\frac{\gamma-1}{\gamma}\right)\frac{M_j}{[1+(\gamma-1)/2(M_j^2)]^{1/2}}\left(\frac{\rho_2}{\rho_1}\right)$$
$$\frac{(\phi_m+\phi_{cf_0}) - \theta[(\phi_m+\phi_{cf_0}) - (\phi_i+\phi_f)]}{[1+(\gamma-1)/2(M_2^2)]} \tag{3}$$

For $S_j = S$, the relation between M_2 and θ becomes[3]

$$M_2 = M_j\left[\theta\left(1+\frac{\gamma-1}{2}M_j^2\right) - \frac{\gamma-1}{2}M_j^2\right]^{-1/2} \tag{4}$$

Hence, for a given jet Mach number M_j, all the functions of M_2 appearing on the right-hand side of Eq. (3) may be calculated as functions of θ; that is, the differential equation (3) is of the form $d\theta/d\tau = f(\theta)$. It has been solved numerically by the method of Runge-Kutta with the initial condition $\theta_{(\tau=0)} = \theta_i$. With $M_j = 2$, for instance, it is found that the final temperature ratio θ_f is about 2 for diatomic gases and 3 for monoatomic gases. These values are only slightly smaller than those computed in Ref. 3, where θ_f was computed for the case the tube walls had also reached their thermal equilibrium.

The time τ^* taken for the driven gas to reach 99% of its total temperature increase if given in Fig. 3. It can be seen that, with a higher jet Mach number, the number of oscillations needed to approach θ_f is smaller. But in all cases considered here, this number is inferior to 10. As an example, the gas in a 10-mm long HS-tube driven with a helium jet at Mach 2 will heat up to 900°K in about 0.2 msec. This corresponds to a heating rate of 3×10^6K/sec.

It is concluded that very high heating rates of the driven gas can be obtained in a HS-tube. The results suggest that, in addition to the application studied in Refs. 1 and 2, HS-tubes could find interesting applications in the field of high-temperature physics and chemistry.

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